

MATHWINGS

Effects on Student Mathematics Performance

Nancy A. Madden
Robert E. Slavin
Kathleen Simons

Johns Hopkins University
- and -
Success for All Foundation

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The Center

Every child has the capacity to succeed in school and in life. Yet far too many children, especially those from poor and minority families, are placed at risk by school practices that are based on a sorting paradigm in which some students receive high-expectations instruction while the rest are relegated to lower quality education and lower quality futures. The sorting perspective must be replaced by a “talent development” model that asserts that all children are capable of succeeding in a rich and demanding curriculum with appropriate assistance and support.

The mission of the Center for Research on the Education of Students Placed At Risk (CRESPAR) is to conduct the research, development, evaluation, and dissemination needed to transform schooling for students placed at risk. The work of the Center is guided by three central themes — ensuring the success of all students at key development points, building on students’ personal and cultural assets, and scaling up effective programs — and conducted through seven research and development programs and a program of institutional activities.

CRESPAR is organized as a partnership of Johns Hopkins University and Howard University, in collaboration with researchers at the University of California at Santa Barbara, University of California at Los Angeles, University of Chicago, Manpower Demonstration Research Corporation, University of Memphis, Haskell Indian Nations University, and University of Houston-Clear Lake.

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Abstract

Constructivist approaches to mathematics instruction based on the standards of the National Council of Teachers of Mathematics (NCTM) have been widely advocated and are expanding in use. However, many educators express a need for constructivist approaches that provide specific student materials, assessments, teachers' manuals, professional development, and other supports to enable a broad range of teachers to succeed with a broad range of children. MathWings was designed to accomplish this goal. MathWings provides a practical, comprehensive approach based on the NCTM standards.

Three evaluations have examined the impact of MathWings. One, involving four rural Maryland schools, found substantially greater gains on the mathematics sections of the Maryland School Performance Assessment Program for MathWings students than for the rest of the state. The four pilot schools, which were much more impoverished than the state as a whole, started far below state averages but ended up above the state average. The second study, in San Antonio, Texas, also found substantial gains on the Texas Assessment of Academic Skills math scale in grades 3-5 from the year before the program began to the end of the first implementation year. The third study found substantial gains on the CTBS mathematics concepts and applications scale for grades 4-5 (but not 3) in a Palm Beach County, Florida school. This report updates and extends *MathWings: Early indicators of effectiveness* (Madden, Slavin, & Simons, 1997).

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Introduction

The teaching of mathematics in the early elementary grades is in the midst of a revolution. This revolution goes under many names, but the name most often attached to it is constructivism. Constructivist mathematics teaching emphasizes *understanding* rather than algorithms (Carpenter et al., 1994; Davis, Maher, & Noddings, 1990). It begins with problem solving and “authentic” complex tasks, rather than building up from arithmetic. For example, children in kindergarten can figure out how many busses are needed to get the class to a picnic long before they learn any division algorithm. They can figure out how to share a pizza fairly long before they learn formal representations of fractions. Constructivist methods make extensive use of cooperative learning, projects, and integrated thematic units. They use many external representations of mathematical ideas, such as base-ten blocks, pictures, and stories. Constructivist theories see the learner as active, intrinsically motivated, and possessing background knowledge and experience that can and must be taken into account in instruction (Paris & Byrnes, 1989). In this view, the task of mathematics instruction is more to introduce students to symbolic representations of concepts they already possess than to teach completely new ideas. For example, children arrive in kindergarten knowing a great deal about combining and separating, more and less, halves and wholes, and so on. Constructivist teaching methods recognize and build on this knowledge, emphasizing discovery, reflection, multiple solutions, and explanation of learning processes by children themselves (Resnick, 1992).

The broad influence of standards developed by the National Council of Teachers of Mathematics, the acceptance of closely related standards by many states, and the development of performance tests increasingly used for state accountability purposes, have all added significantly to the press for more constructivist teaching in elementary schools. The stakes for schools and students are rising. State-of-the-art state mathematics assessments require students to solve complex, non-routine problems and to explain their thinking processes.

These new standards and assessments create significant opportunities for reform in the teaching of mathematics at all levels. Yet they also create a serious danger. Studies of new performance assessments are finding that poor and minority students are scoring worse on these assessments (relative to middle-class students) than they do on traditional standardized measures (Shavelson, Baxter, & Pine, 1992). If anything, this problem is likely to become worse. Moving from traditional to constructivist teaching requires a substantial investment in top-quality professional development. Middle class school districts are likely to be able to make such investments and to be able to hire elementary teachers who are already reasonably proficient in mathematics. Impoverished schools are less likely to have teachers who are up-to-date in new conceptions of mathematics or to be able to provide the months of inservice often required to enable even good teachers with strong interests and backgrounds in mathematics to internalize learner-centered teaching in mathematics.

A number of new approaches to mathematics curriculum and instruction have been developed for elementary schools to help them move toward constructivist conceptions of learning. Examples include Conceptually Based Instruction (Hiebert & Wearne, 1993), Cognitively Guided Instruction (Carpenter & Fennema, 1992), Supporting Ten-Structured Thinking (Fuson, 1992), and QUASAR (Stein & Lane, 1995). These and other methods are expanding in use in elementary schools. Yet there is still a need for further development and research directed at creating practical constructivist methods capable of being used on a large scale by all teachers, not only those with particular interests and backgrounds in mathematics. Many projects have shown success on a limited scale at introducing constructivist methods in elementary schools, including those serving many students placed at risk (see, for example, Fuson, 1992; Jamar, 1995; Stein & Lane, 1995; Campbell, Cheng, & Rowan, 1995; Carpenter Fennema, Peterson, Chiang, & Loef, 1989). However, mathematics instruction, especially in urban elementary classrooms, remains overwhelmingly algorithmic, teacher-centered, and traditional.

The goal of reform in elementary mathematics must be to provide deep understanding of mathematical ideas for *all* students, not just for those fortunate enough to have teachers with extraordinary skills and interests in mathematics. Mathematics for *all* will require approaches very different from those needed to demonstrate on a small scale that students can learn in new ways. It will require the development of new curricula and school support structures capable of ensuring that every elementary teacher, even those in high-poverty, underfunded schools, will be able to enable students to be strategic, flexible, self-aware, and motivated problem solvers in mathematics.

This report describes three evaluations of a mathematics instruction program designed to make constructivist mathematics instruction practical and successful for a broad range of children and teachers in high poverty schools. This program, called MathWings, is part of a comprehensive school reform approach called Roots and Wings (Slavin, Madden, & Wasik, 1996), which was funded by New American Schools. Roots and Wings adds MathWings as well as social studies and science programs to a reading, writing, and language arts program called Success for All (Slavin, Madden, Dolan, & Wasik, 1996).

The development of MathWings drew heavily on the experience of developing Success for All, although the curricular approaches are quite different. Both strategies emphasize well-structured student materials, frequent assessment, cooperative learning, effective classroom management methods, and extensive teacher training and followup. The idea is to improve the instructional strategies of all teachers, whether or not they are experts in mathematics, but to build in enough flexibility to allow the best teachers to go further.

This report describes MathWings and presents the first three evaluations of the program's effects on student mathematics achievement.

MathWings: Program Description

MathWings is based on the standards of the National Council of Teachers of Mathematics. A program to prepare students for mathematics in the twenty-first century needs to actively involve students in the conceptual development and practical application of their mathematics skills. MathWings reflects a balance of solid mathematical conceptual development, problem solving in real world applications, and development of necessary mathematics skills.

Students enter school with a great deal of mathematical knowledge. They know about combining and separating, halves and wholes, and so on. What they need is a bridge between their pre-existing knowledge and the formal representation of this knowledge in mathematical symbols. This requires the use of manipulatives, demonstrations, and discovery to help students build mathematical understanding. MathWings uses cooperative learning at all age levels while incorporating problem solving in real situations, skill practice and reinforcement for efficiency in application, calculator use, alternative assessments, writing, connections to literature and other disciplines, and application to the students' world and personal experiences. Although students help each other learn, they are always individually accountable for their own learning, and are frequently assessed on their progress in understanding and using math (see Slavin, 1995).

Major Components

The *NCTM Curriculum and Evaluation Standards* (1989) advocate emphasizing problem solving rather than rote calculation with algorithms. MathWings lessons involve the students in problem solving in “real” situations to give validity and purpose to their mathematics explorations, and in daily problem solving as part of the routine of math class. MathWings lessons also make connections to literature, science, art, and other subjects as well as the students' world and personal experiences to provide this real world problem solving context.

Another strand of the Standards is mathematical reasoning. Students develop their ability to think through and solve mathematical problems when they use manipulatives to develop concepts and then represent what is actually happening with symbols. MathWings units are constructed to develop concepts from the concrete to the abstract so that each step of the reasoning is clarified. The Standards also promote the use of calculators for developing concepts and exploring advanced problem solving situations rather than checking answers or replacing skills and mental math. MathWings students use calculators in this way to increase both their mathematical reasoning skills and the scope and complexity of the problems they can solve, and to focus their energy on mathematical reasoning rather than mere mechanical calculation.

The Standards emphasize communication, both oral and written, to clarify, extend, and refine the students' knowledge. MathWings students constantly explain and defend their solutions orally throughout the lessons, and regularly write in their logbooks. This emphasis on communication extends to the assessments as well. The Standards suggest the use of alternative assessments which incorporate communication as well as calculation. MathWings units involve the students in many different types of assessment. The students complete concept checks in which they explain their thinking as they solve problems after every few lessons. They work on performance tasks at the end of each unit to use the skills they have learned to solve practical real world situations and explain and communicate about their thinking. Teacher observations of students at work with manipulatives, collecting data, and other activities, as well as their written and oral communications, are all used to assess their understanding.

The use of cooperative learning in MathWings is based on years of research regarding effective strategies for classroom instruction. This research has shown that the cognitive rehearsal opportunities presented by cooperative learning, as well as the opportunities for clarification and reteaching for students who do not catch a concept immediately, have positive effects on academic achievement. Research has also shown that using cooperative learning in the classroom can have positive effects on inter-ethnic relationships, acceptance of mainstreamed, academically handicapped students, student self-esteem, liking of others, and attitudes toward school and teachers (Slavin, 1995). In cooperative learning, students work together to learn; the team's work is not done until all team members have learned the material being studied. This positive interdependence is an essential feature of cooperative learning.

Research has identified three key components which make cooperative learning strategies effective: team recognition, individual accountability, and equal opportunities for success. In MathWings, as in other Student Team Learning strategies (Slavin, 1994), students work in four-member, mixed-ability teams. Teams may earn certificates and additional means of recognition if they achieve at or above a designated standard. All teams can succeed because they are working to reach a common standard rather than competing against one another. The team's success depends on the individual learning of all team members; students must make sure that everyone on the team has learned, since each team member must demonstrate his or her knowledge on an individual assessment. Students have an equal opportunity for success in MathWings because they contribute points to their teams by improving over their own individual performance, by bringing in their homework, and by meeting particular behavior goals set by the teacher. Students who are typically seen as low achievers can contribute as many points to the team as high achievers.

The MathWings program is designed to use the calculator as a tool, not a crutch. Calculators enable the students to explore and demonstrate concepts in an appealing way. Students discover that they need to check their calculator answers for accuracy since the calculator is only as accurate as the information and process that is keyed into it. Thus, students develop their skills in estimating and predicting outcomes. Students also spend more time actually thinking about math and the processes that will most efficiently solve a given problem, rather than focusing completely on tedious and lengthy calculations. Because of the speed of calculation with calculators, students are more willing to try several approaches to solving a problem situation or to reevaluate their answers and try a different method of solution. Finally, calculators build students' confidence in mathematics as they receive much positive reinforcement from correct solutions. This leads, in turn, to a greater willingness to tackle more challenging mathematical situations in the belief that they have the ability and the tools to solve them.

Manipulative use is a basic building block of the MathWings program at all levels. Students construct understanding and develop original methods for solving problems using manipulatives. As they work with manipulatives and discuss and defend their thinking, they gradually make the concepts their own. Once a problem can be solved with manipulatives, students draw a picture and then write a number sentence to represent what was happening with the manipulatives as they solved the problem. This gradual progression from concrete to pictorial to abstract provides a solid foundation of understanding upon which the students can build. Every method or algorithm can be understood, and even reinvented, with manipulatives, thus replacing rote learning of algorithms with understanding of concepts and ways to efficiently apply them. Once the concepts have been firmly established and students understand how the algorithms work, they move away from using concrete manipulatives. However, manipulatives can be revisited at any time to remediate or extend a concept as needed.

Most MathWings whole-class units have a literature connection which is an integral part of the concept development. Literature provides a wonderful vehicle for exploring mathematical concepts in meaningful contexts demonstrating that mathematics are an integral part of human experience. The use of literature incorporates the affective elements and demonstrates the aesthetic aspect of mathematics. Finally, the use of literature encourages students to pose problems from real and imaginary situations and to use language to communicate about mathematics.

MathWings involves the students in daily routines for skill practice and reinforcement to facilitate efficiency in calculation and application. Once the students have mastered the facts and basic algorithms, they become tools for the students to use as they develop concepts and problem solving. These daily routines include practice and weekly timed tests to encourage mastery of the basic facts, and then daily practice problems at varying difficulty levels to provide for fluency in the use of the essential algorithms.

Daily Schedule

MathWings is composed of two kinds of units: Action Math units, which are whole class units, and Power Math units, which are individualized units. Action Math units are 2 to 6 weeks in length and explore and develop concepts and their practical applications. One-week Power Math units are interspersed between whole-class units and provide time for students to individually practice previous skills or explore more accelerated skills.

There is a frame around the lesson every day. This frame is provided by the daily routines, which are efficient ways to provide for team management, problem solving, and fluency of skills. Daily routines are part of every Action Math or Power Math lesson.

Every day, students spend at least 60 minutes in their mathematics class. Daily lessons consist of three components: Check-In, Action Math or Power Math, and Reflection.

The first 15-minute segment is Check-In. Check-In is an efficient class start-up routine in which team members regularly complete one challenging real world problem individually and then come to a team consensus about their problem solving strategies and solutions. They also complete a facts study process twice a week, and check homework briefly every day.

The next 40 minutes in either Action Math or Power Math is the heart of the lesson. When the class is doing an Action Math unit, the lesson involves the students in active instruction, team work, and assessment. During active instruction the teacher and students interact to explore a concept and its practical applications and related skills. The teacher may present a challenging problem for students to explore with manipulatives to construct a solution, may challenge the teams to use prior knowledge to discover a solution, or may ask the teams to find a pattern to develop a rule.

During team work the students first solve a problem together to develop their understanding of concepts. A team member is chosen randomly to share his or her ideas with the class. Then students individually practice similar problems, with teammates available for support as needed. The team members check answers and explanations with each other and rehearse to be sure every team member can explain them.

At the end of the team work, there is another brief feedback opportunity. The teacher randomly chooses a team member to share the ideas or solutions of the team, and to explain their thinking. This enables the teacher to assess the understanding of the group as a whole and ensures that teammates are invested in making sure that all members of the team are mastering the concepts.

The final portion of an Action Math lesson is assessment. One or more brief problems are used as a quick individual assessment of mastery of the concept or skill explored in the lesson.

When the class is doing a one-week Power Math unit, the 40-minute heart of the lesson involves each student in remediating, refining, or accelerating his/her skills. Students work at their own pace on the skill they need to practice, completing check-outs and mastery tests successfully to move to another skill they need to practice. Students who have mastered the basic skills explore accelerated units at their own pace. The teacher teaches mini-lessons to small groups of students (working on the same skills) gathered from various teams while the other students continue to work individually.

The last five-minute segment of class is Reflection. This is an efficient routine used to bring closure to the class time. During Action Math units, reflection involves a quick summary of the key concepts by the teacher. During both Action Math units and Power Math units, a short entry is written in the MathWings Logbook in response to a writing prompt about the lesson, and homework sheets are passed out.

All students should not only be given the opportunity to establish a solid foundation in mathematics, but also the opportunity to extend and stretch their knowledge and experience in mathematics. Thus, a program of mathematics should include a structure to accommodate a diversity of abilities and background mathematical knowledge, while ensuring that *all* students experience the depth, breadth, and beauty of mathematics. The MathWings curriculum incorporates this philosophy in its development.

Research on MathWings

Data on the achievement outcomes of MathWings are available for six early pilot schools — four in rural Maryland, one in San Antonio, Texas, and one in Palm Beach County, Florida.

St. Mary's County, Maryland

The pilot schools for all of Roots & Wings, including MathWings, are four schools in and around Lexington Park, a rural community in Southern Maryland. The four schools are by far the most impoverished schools in the district; on average, 48% of their students qualify for free lunch. The schools began implementing the reading aspects of Roots & Wings in 1993-94, and then began to phase in MathWings in grades 3-5 in 1994-95. By 1995-96, all teachers in grades 3-5 were using MathWings, and piloting in grades 1-2 began in 1996-97.

Because there were no schools in St. Mary's County comparable in poverty or prior achievement to the Roots & Wings schools, an experimental-control comparison within the district could not be carried out. Instead, test score gains over time in the Roots & Wings

schools were compared to those in the state as a whole. The test is the Maryland School Performance Assessment Program, or MSPAP, a state-of-the-art performance assessment in which students are asked to solve complex problems, set up experiments, write in several genres, and so on. The MSPAP uses matrix sampling, which means that different children take different parts of a broad, comprehensive test. Scores are reported in terms of percentages of children achieving at high levels labeled “satisfactory” and “excellent” in each school. In elementary schools, only third and fifth graders are assessed.

The results for the MSPAP mathematics scales are summarized in Figures 1 and 2. Figure 1 shows that third graders in the four MathWings pilot schools started off far below the state average. By 1995, they had essentially caught up to the state average and, in 1996, exceeded it. In 1997, the scores slipped slightly behind those of the state, but were still substantially ahead of baseline, both in absolute terms and relative to state means. Exactly the same pattern was found in fifth grade, where, by 1996, the MathWings pilot schools also had higher scores than the state average, and fell slightly below in 1997.

San Antonio, Texas

One of the first pilot schools for MathWings outside of Maryland was Lackland City Elementary School in the Northside Independent School District in San Antonio, Texas. Lackland City had implemented the Success for All reading program in 1994-95, and then began to pilot MathWings in 1995-96. Lackland City is one of the most impoverished schools in its district; 86% of its students qualify for free lunch. A majority of its students are Latino (78%), with a high proportion categorized as Limited English Proficient.

Students in Texas are tested annually on the Texas Assessment of Academic Skills, or TAAS. Scores are reported in terms of the percentages of children passing the TAAS in each subject. Figure 3 shows the TAAS mathematics gains for Lackland City in grades 3, 4, and 5 from 1994-95 (just before the program began) to 1995-96, the first implementation year.

As Figure 3 illustrates, students in all three grades made substantial gains on the TAAS, in comparison to the previous cohorts of students in the same school. Percent passing more than doubled in third grade, from 36.7% to 76.7%. Fourth grades did almost as well, increasing from 34.2% to 60.9% passing, and fifth grades increased from 52.9% to 86.8% passing. Although Lackland City is far more impoverished than its district average (86% free lunch vs. 42% for the district), its students had TAAS passing rates higher than the district average in third grade (76.7% to 73.7%) and fifth grade (86.8% to 81.5%), though not in fourth grade (60.9% to 75.5%).

Figure 1
Mathematics Performance, Maryland School Performance Assessment Program, 1993-97
MathWings Pilot Schools vs. State of Maryland
Grade 3

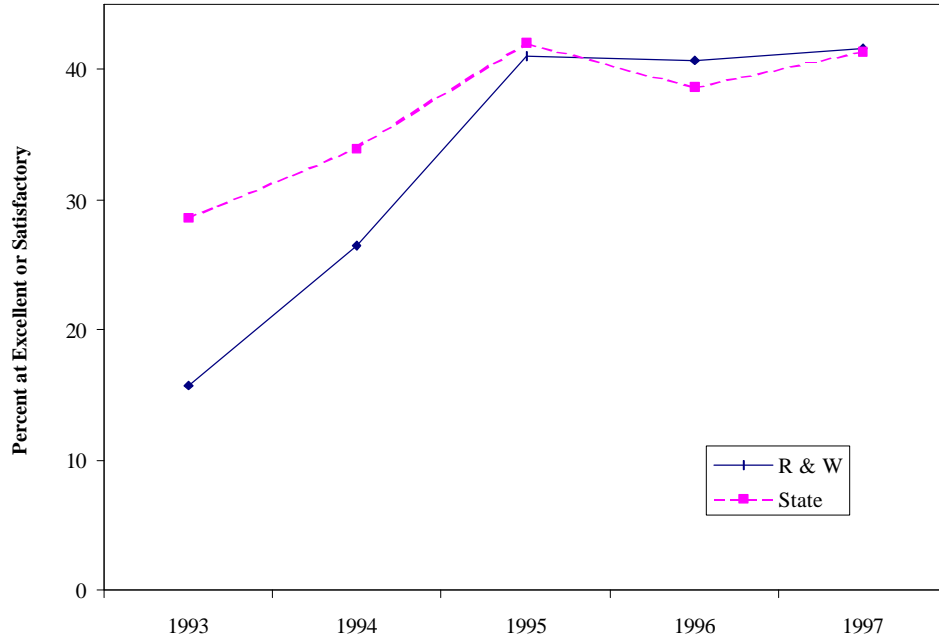


Figure 2
Mathematics Performance, Maryland School Performance Assessment Program, 1993-97
Math Wings Pilot Schools vs. State of Maryland
Grade 5

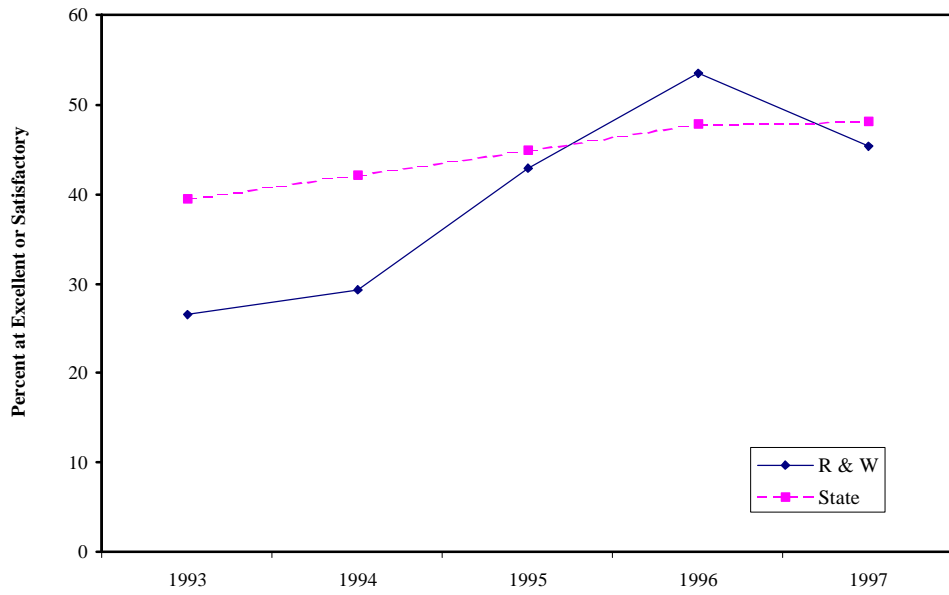


Figure 3
Percent of Students Passing TAAS Math in 1995 (Pre) and 1996 (Post), Lackland City Elementary School

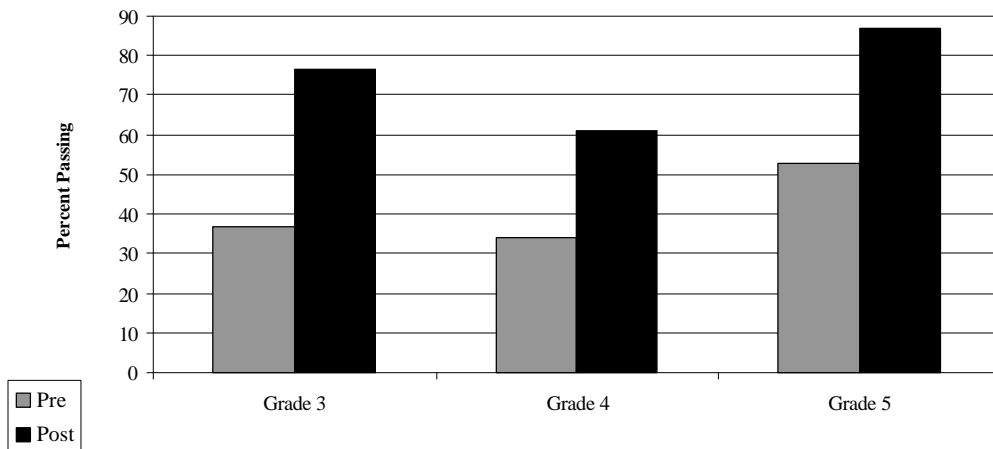
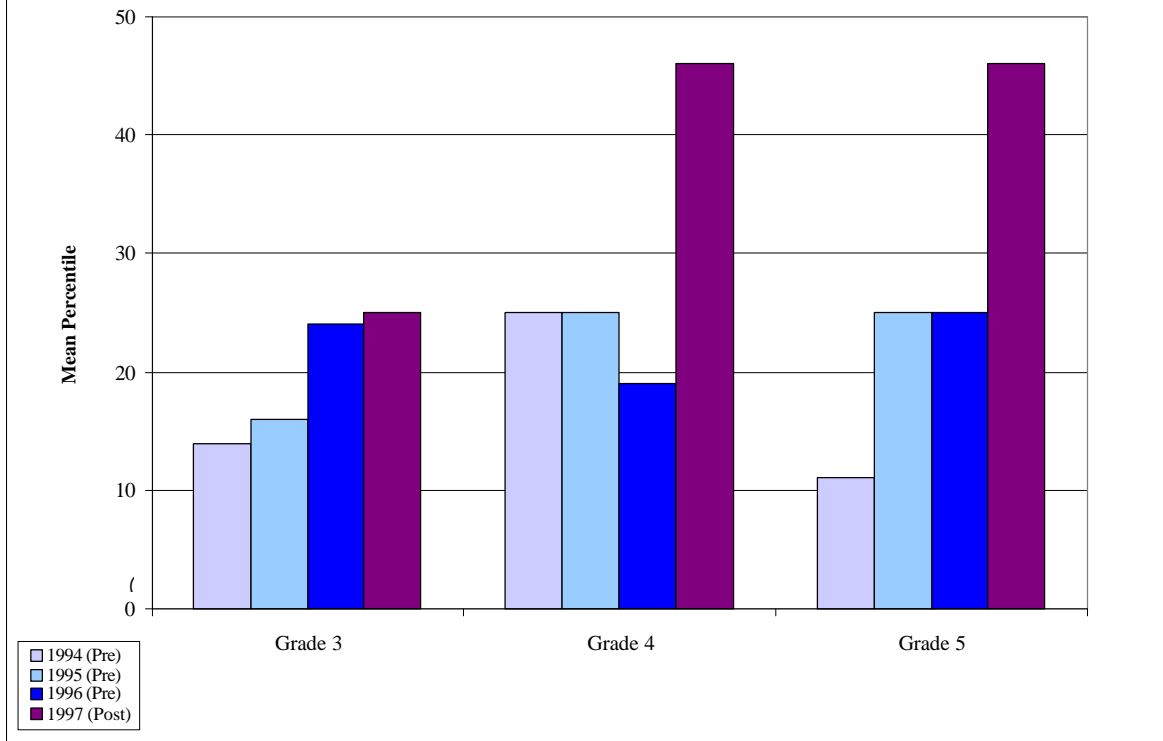


Figure 4
CTBS Math Concepts & Applications, Mean Percentile Rank (PR), 1994-97
Lincoln Elementary School



Palm Beach County, Florida

Another early pilot site for MathWings was Lincoln Elementary School in West Palm Beach, Florida. Lincoln began implementation of Success for All in 1993-94, and began its MathWings pilot in 1996-97. Like Lackland City, Lincoln is one of the most impoverished schools in its district; 100% of its students are African American, and more than 90% qualify for free lunch.

Florida elementary schools take the Comprehensive Test of Basic Skills mathematics concepts and applications scale in grades 3-5. Percentage scores for the three years preceding MathWings implementation and for the first year following are shown in Figure 4.

Figure 4 shows substantial gains in grades 4 and 5, a gain of 27 percentile points in grade 4 and 21 percentage points in grade 5. These gains put this very impoverished school nearly at grade level (46th percentile), and ahead of the district's own math-science-technology magnet school. Grade 3 gains, however, were slight (only one percentage point).

Conclusion

Trends on state accountability measures for six pilot schools in three districts show substantial gains due to implementation of MathWings. In all three districts, high-poverty schools initially performing significantly below district or state averages reached or exceeded these averages after implementing MathWings.

There is much more to be done in the evaluation of MathWings. None of the evaluations reported here used control groups, and it is possible in all three cases that at least part of the gains in mathematics performance is due to implementation of the Success for All reading program, implemented in all of these MathWings pilot schools 1-3 years earlier. Experimental-control comparisons among schools that have all implemented Success for All are currently underway. However, the dramatic gains seen in the six pilot schools are unlikely to have been entirely due to the reading program or to other factors; in all districts assessed, baseline scores reported here would already reflect the reading implementations, and the largest gains were seen in the year when MathWings was implemented.

These results demonstrate that schools serving many children in poverty can substantially accelerate the mathematics achievement of their students using an approach tied to NCTM standards but developed to be practical for a broad range of teachers and effective for a broad range of students.

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